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A developed Best-Worst method to solve multi-criteria decision-making problems under intuitionistic fuzzy environments

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Abstract: In real-world decision-making problems, we often face multiple criteria that should be considered in the decision process. Also, the collected data are often non-crisp and reported with a degree of hesitation. In this study, using intuitionistic fuzzy numbers that can model these non-crisp and hesitant data, we propose a simple yet effective method for solving real-world multi-criteria decision-making problems. We applied our proposed approach to the intuitionistic fuzzy Best-Worst Method, which is one of the famous techniques for solving multi-criteria decision-making problems. However, the proposed method can be easily generalized to the other multi-criteria decision-making methods in intuitionistic fuzzy environments, too. Finally, the ability of the proposed approach to solve these types of problems is shown by an illustrative example.

Keywords: Intuitionistic fuzzy numbers, Multi-criteria decision making; Best-Worst method; Hesitation degree, Decision level
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1 Introduction

Multi-criteria decision-making (MCDM) is a collection of various decision-making methods to help decision-makers make the best decision in the presence of multiple criteria and the specific conditions that should consider in the problem. Many approaches proposed to solve MCDM problems so far. Among these methods, for example, we can mention the AHP (Analytic Hierarchy Process) method [20-22], ANP (Analytic Network Process) method [23,24], ELECTRE (Elimination and Choice Expressing REality) method [19], TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method[9, 30], VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method [8,15], SWARA (step-wise weight assessment ratio analysis) method [11], DEMATEL (DEcision-MAking Trial and Evaluation Laboratory) method [28,29], and PROMETHEE

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(Preference Ranking Organization METHod for Enrichment of Evaluations) method [4,5]. But, developing the methods with higher accuracy and more practical is in the attention of researchers yet. Especially in real-world applications, applying an efficient model to solve MCDM problems is vital.

The Best-Worst method (BWM) proposed by Rezaei [16] in 2015 is one of the most famous and applicable methods in the MCDM field. The Best-Worst method is a more advanced version than the AHP to solve MCDM problems. It has better consistency and is more user-friendly. However, in real-world applications that the data are often non-crisp and uncertain, we cannot use the traditional BWM.

Intuitionistic fuzzy numbers (IFNs) first proposed by Burillo [6] can efficiently model non-crisp and uncertain data. They are a developed version of fuzzy numbers that consider the hesitation concept in modeling data. Lots of papers about the application of intuitionistic fuzzy numbers in modeling MCDM problems are published so far. For example, Abdullah et al. [1] proposed an interval-valued intuitionistic fuzzy DEMATEL method combined with Choquet integral for sustainable solid waste management. Mishra et al. [14] developed an intuitionistic fuzzy divergence measure-based ELECTRE method for the performance of cellular mobile telephone service providers. Also, Rouyendegh [18] defined a new intuitionistic fuzzy ELECTRE method to handle more complicated problems. In another work, Rouyendegh et al. [17] proposed an intuitionistic fuzzy TOPSIS method for the green supplier selection problem. Krishankumar et al. [12] introduced a developed VIKOR method under intuitionistic fuzzy environments to solve the personnel selection problem. Krasan and Kahraman [10] suggested a new intuitionistic fuzzy DEMATEL – ANP – TOPSIS integrated approach for freight village location selection.

In this paper, we develop the Best-Worst method to solve MCDM problems under the intuitionistic fuzzy environments. For this purpose, we first Substitute the intuitionistic fuzzy numbers (that applied in the Best-Worst method to compare criteria) with the assigned parametric index that was proposed by Shakouri et al. [25]. So, the intuitionistic fuzzy Best-Worst model converts to a parametric Best-Worst model. Finally, by appropriate selection of the parameters by the decision-maker, we solve the model. The parameters are the decision level and the hesitation degree. Therefore, the decision-maker can meaningfully apply his/her opinion to the decision process.

The remainder of this research is organized as follows. In Section 2, some basic concepts about intuitionistic fuzzy sets (IFSs), intuitionistic fuzzy numbers (IFNs), and the Best-Worst method are proposed. In Section 3, we propose our intuitionistic fuzzy Best-Worst method to solve MCDM problems under intuitionistic fuzzy environments. In Section 4, by an illustrative example, we demonstrate the ability of our proposed method to solve intuitionistic fuzzy MCDM problems using the developed Best-Worst method. Finally, Section 5 concludes the paper.

2 Preliminaries

In this section, some basic concepts about intuitionistic fuzzy sets (IFSs), intuitionistic fuzzy numbers (IFNs), and the Best-Worst method are proposed.

2.1 Intuitionistic fuzzy sets

Definition 2.1. [2,3] Let X be a fixed universe. We can present an intuitionistic fuzzy set (IFS) $\tilde{I}$ in X as $\{\langle x, \mu_{\tilde{I}}(x), v_{\tilde{I}}(x)\rangle | x \in X\}$, that $\mu_{\tilde{I}}(x)$ is the degree of membership function from X to $[0,1]$, and $v_{\tilde{I}}(x)$ is the degree of non-membership function from X to $[0,1]$. Moreover, $0 \leq \mu_{\tilde{I}}(x), v_{\tilde{I}}(x) \leq 1$, and $0 \leq \mu_{\tilde{I}}(x) + v_{\tilde{I}}(x) \leq 1$.

Definition 2.2. [2,3] We can define the hesitation function of an IFS $\tilde{I}$ as

$$\pi_{\tilde{I}}(x) = 1 - \mu_{\tilde{I}}(x) - v_{\tilde{I}}(x)$$

Where $0 \leq \pi_{\tilde{I}}(x) \leq 1$.

The $\pi_{\tilde{I}}(x)$ shows the degree of hesitation or indeterminacy of the element x to belonging or not belonging to $\tilde{I}$. Also, if $v_{\tilde{I}}(x) = 1 - \mu_{\tilde{I}}(x)$, then $\pi_{\tilde{I}}(x) = 0$, and so, $\tilde{I}$ defines a fuzzy set. So, fuzzy sets are special cases of IFSs.

2.2 Intuitionistic fuzzy numbers

Definition 2.3. [13] In the set of real numbers $\mathbb{R}$, we can define an IFN $\tilde{I} = (\mu_{\tilde{I}}(x), v_{\tilde{I}}(x))$ with the following membership and non-membership functions, respectively.

$$\mu_{\tilde{I}}(x) = \begin{cases} k_{\tilde{I}}(x) & a \leq x \leq b, \\ w & b \leq x \leq c, \\ l_{\tilde{I}}(x) & c \leq x \leq d, \\ 0 & \text{otherwise} \end{cases}$$

$$v_{\tilde{I}}(x) = \begin{cases} m_{\tilde{I}}(x) & e \leq x \leq \mathrm{f}, \\ u & \mathrm{f} \leq x \leq \mathrm{g}, \\ n_{\tilde{I}}(x) & \mathrm{g} \leq x \leq \mathrm{h}, \\ 1 & \text{otherwise} \end{cases}$$

$k_{\tilde{I}}$ and $n_{\tilde{I}}$ are non-decreasing and continuous functions from $\mathbb{R}$ to [0,1], and $l_{\tilde{I}}$ and $m_{\tilde{I}}$ are non-increasing and continuous functions from $\mathbb{R}$ to [0,1]. Moreover, $e \leq a, f \leq b \leq c \leq \mathrm{g}, d \leq \mathrm{h}$. Also, w is the maximum degree of $\mu_{\tilde{I}}(x)$ and u is the minimum degree of $v_{\tilde{I}}(x)$. (Figure 1)

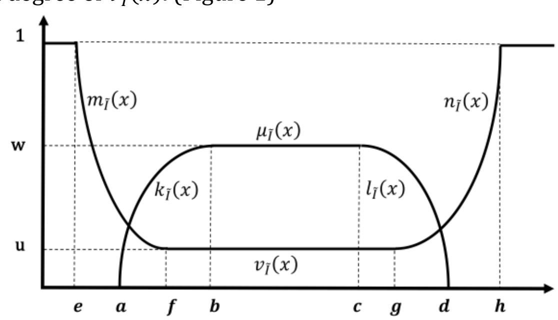

Figure 1. Representation of an IFN $\tilde{I} = (\mu_{\tilde{I}}(x), v_{\tilde{I}}(x))$

Definition 2.4. An IFN $\tilde{I}$ is normal if $w = 1$ and $u = 0$.

Definition 2.5. [25] A Trapezoidal IFN $\tilde{I}$ is an IFN with the following membership function $\mu_{\tilde{I}}$ and non-membership function $v_{\tilde{I}}$.

$$\mu_{\tilde{I}}(x) = \begin{cases} \dfrac{w(x-a)}{b-a} & a \leq x \leq b, \\ w & b \leq x \leq c, \\ \dfrac{w(d-x)}{d-c} & c \leq x \leq d, \\ 0 & \text{otherwise} \end{cases}$$

$$v_{\tilde{I}}(x)=\begin{cases}\dfrac{(u-1)(x-\mathrm{e})}{\mathrm{f}-\mathrm{e}}+1 & \mathrm{e}\le x\le \mathrm{f},\\ u & \mathrm{f}\le x\le \mathrm{g},\\ \dfrac{(u-1)(x-\mathrm{h})}{\mathrm{g}-\mathrm{h}}+1 & \mathrm{g}\le x\le \mathrm{h},\\ 1 & \text{otherwise}\end{cases}$$

We present a Trapezoidal IFN $\tilde{I}$ by $\tilde{I}=\langle(a,b,c,d;w),(\mathrm{e},\mathrm{f},\mathrm{g},\mathrm{h};u)\rangle$. (see Figure 2)

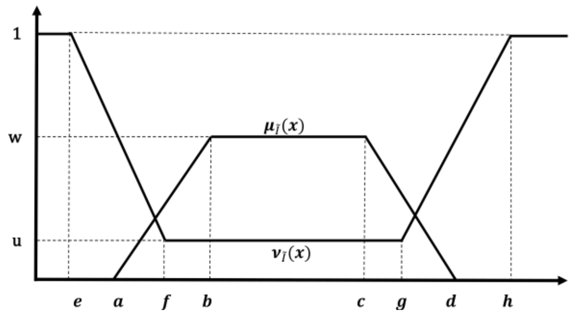

Figure 2. A Trapezoidal IFN $\tilde{I}=\langle(a,b,c,d;w),(\mathrm{e},\mathrm{f},\mathrm{g},\mathrm{h};u)\rangle$

From definition 2.4, for a Trapezoidal IFN if we set $w=1$ and $u=0$, we will have a normal Trapezoidal IFN. Also, if we set $b=c$ and $f=g$, we will have a Triangular IFN.

Definition 2.6. [3] We can define the cut sets of an IFN $\tilde{I}=(\mu_{\tilde{I}}(x),v_{\tilde{I}}(x))$ as follow.

α-cut set of $\tilde{I}$: $\tilde{I}_\alpha=\{x|\mu_{\tilde{I}}(x)\ge\alpha\},\ 0\le\alpha\le w$

β-cut set of $\tilde{I}$: $\tilde{I}_\beta=\{x|v_{\tilde{I}}(x)\le\beta\},\ u\le\beta\le 1$

(α,β)-cut set of $\tilde{I}$: $\tilde{I}_{\alpha,\beta}=\{x|\mu_{\tilde{I}}(x)\ge\alpha, v_{\tilde{I}}(x)\le\beta\},\ 0\le\alpha\le w, u\le\beta\le 1, 0\le\alpha+\beta\le 1$

Definition 2.7. [25] From definition 2.6, the cut sets of a Trapezoidal IFN $\tilde{I}=\langle(a,b,c,d;w),(\mathrm{e},\mathrm{f},\mathrm{g},\mathrm{h};u)\rangle$ will define as the following closed intervals.

$$\tilde{I}_\alpha=\{x|\mu_{\tilde{I}}(x)\ge\alpha\}=\left[a+\frac{\alpha}{w}(b-a),d+\frac{\alpha}{w}(c-d)\right]$$

$$\tilde{I}_\beta=\{x|v_{\tilde{I}}(x)\le\beta\}=\left[\mathrm{e}+\frac{1-\beta}{1-u}(\mathrm{f}-\mathrm{e}),\mathrm{h}+\frac{1-\beta}{1-u}(\mathrm{g}-\mathrm{h})\right]$$

$$\tilde{I}_{\alpha,\beta}=\{x|\mu_{\tilde{I}}(x)\ge\alpha,v_{\tilde{I}}(x)\le\beta\}=\left[a+\frac{\alpha}{w}(b-a),d+\frac{\alpha}{w}(c-d)\right]\cap\left[\mathrm{e}+\frac{1-\beta}{1-u}(\mathrm{f}-\mathrm{e}),\mathrm{h}+\frac{1-\beta}{1-u}(\mathrm{g}-\mathrm{h})\right]$$

Definition 2.8. [7] The arithmetic operations on two arbitrary trapezoidal IFNs

$\tilde{I}_1=\langle(a_1,b_1,c_1,d_1;w_1),(e_1,f_1,\mathrm{g},h_1;u_1)\rangle$ and $\tilde{I}_2=\langle(a_2,b_2,c_2,d_2;w_2),(e_2,f_2,g_2,h_2;u_2)\rangle$

are as follows.

$$\tilde{I}_1\oplus\tilde{I}_2=\langle(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2;\min\{w_1,w_2\}),(e_1+e_2,f_1+f_2,g_1+g_2,h_1+h_2;\max\{u_1,u_2\})\rangle$$

$$\tilde{I}_1\ominus\tilde{I}_2=\langle(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2;\min\{w_1,w_2\}),(e_1-h_2,f_1-g_2,g_1-f_2,h_1-e_2;\max\{u_1,u_2\})\rangle$$

$$\tilde{I}_1\otimes\tilde{I}_2=\begin{cases}\langle(a_1a_2,b_1b_2,c_1c_2,d_1d_2;\min\{w_1,w_2\}),(e_1e_2,f_1f_2,g_1g_2,h_1h_2;\max\{u_1,u_2\})\rangle & \text{if } \tilde{I}_1>0 \text{ and } \tilde{I}_2>0,\\ \langle(a_1d_2,b_1c_2,c_1b_2,d_1a_2;\min\{w_1,w_2\}),(e_1h_2,f_1g_2,g_1f_2,h_1e_2;\max\{u_1,u_2\})\rangle & \text{if } \tilde{I}_1<0 \text{ and } \tilde{I}_2>0,\\ \langle(d_1d_2,c_1c_2,b_1b_2,a_1a_2;\min\{w_1,w_2\}),(h_1h_2,g_1g_2,f_1f_2,e_1e_2;\max\{u_1,u_2\})\rangle & \text{if } \tilde{I}_1<0 \text{ and } \tilde{I}_2<0\end{cases}$$

$\tilde{I}_1\oslash\tilde{I}_2=$

$$\begin{cases} \langle(a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2; \min\{w_1, w_2\}), (e_1/h_2, f_1/g_2, g_1/f_2, h_1/e_2; \max\{u_1, u_2\})\rangle & \text{if } \tilde{I}_1 > 0 \text{ and } \tilde{I}_2 > 0, \\ \langle(d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2; \min\{w_1, w_2\}), (h_1/h_2, g_1/g_2, f_1/f_2, e_1/e_2; \max\{u_1, u_2\})\rangle & \text{if } \tilde{I}_1 < 0 \text{ and } \tilde{I}_2 > 0, \\ \langle(d_1/a_2, c_1/b_2, b_1/c_2, d_1/a_2; \min\{w_1, w_2\}), (h_1/e_2, g_1/f_2, f_1/g_2, h_1/e_2; \max\{u_1, u_2\})\rangle & \text{if } \tilde{I}_1 < 0 \text{ and } \tilde{I}_2 < 0 \end{cases}$$

$$\lambda\tilde{I}_1 = \begin{cases} \langle(\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1), (\lambda e_1, \lambda f_1, \lambda g_1, \lambda h_1; u_1)\rangle & \text{if } \lambda \geq 0, \\ \langle(\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1), (\lambda h_1, \lambda g_1, \lambda f_1, \lambda e_1; u_1)\rangle & \text{if } \lambda < 0 \end{cases}, \quad \lambda \in \mathbb{R}$$

$$\tilde{I}_1^{-1} = \langle(1/d_1, 1/c_1, 1/b_1, 1/a_1; w_1), (1/h_1, 1/g_1, 1/f_1, 1/e_1; u_1)\rangle$$

Shakouri et al. [25] proposed a new definition of an arbitrary IFN as below. In this definition, the independent variable is on the vertical axis.

Definition 2.9. Let $\tilde{I} = (\mu'_{\tilde{I}}(r), v'_{\tilde{I}}(r))$ is an IFN, so that $\mu'_{\tilde{I}}(r)$ is $\left(\underline{P}(r), \overline{P}(r)\right)$ that is a pair of functions $\underline{P}(r)$ and $\overline{P}(r); 0 \leq r \leq w$ with the following properties:

1) $\underline{P}(r)$ is a bounded monotonic increasing left continuous function.

2) $\overline{P}(r)$ is a bounded monotonic decreasing left continuous function.

3) $\underline{P}(r) \leq \overline{P}(r), 0 \leq r \leq w$.

, and

$v'_{\tilde{I}}(r)$ is $\left(\underline{Q}(r), \overline{Q}(r)\right)$ that is a pair of functions $\underline{Q}(r)$ and $\overline{Q}(r); u \leq r \leq 1$ with the following properties:

1) $\underline{Q}(r)$ is a bounded monotonic decreasing left continuous function.

2) $\overline{Q}(r)$ is a bounded monotonic increasing left continuous function.

3) $\underline{Q}(r) \leq \overline{Q}(r), u \leq r \leq 1$.

According to the definition 2.9, a Trapezoidal IFN can be represented as follows (figure 3).

$\tilde{I} = \left(\mu'_{\tilde{I}}(r), v'_{\tilde{I}}(r)\right)$, where $\mu'_{\tilde{I}}(r) = \left(\underline{P}(r), \overline{P}(r)\right) = \left(a + \frac{r}{w}(b-a), d + \frac{r}{w}(c-d)\right), \ 0 \leq r \leq w$, and

$v'_{\tilde{I}}(r) = \left(\underline{Q}(r), \overline{Q}(r)\right) = \left(\mathrm{e} + \frac{1-r}{1-u}(\mathrm{f}-\mathrm{e}), \mathrm{h} + \frac{1-r}{1-u}(\mathrm{g}-\mathrm{h})\right), u \leq r \leq 1$.

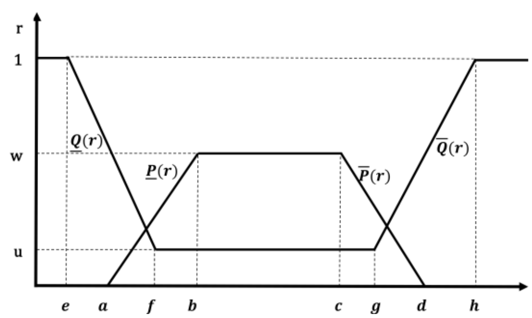

Figure 3. A Trapezoidal IFN.

2.3 Best-Worst method

Multi-criteria decision-making (MCDM) problem is a complicated cognitive process of choosing an alternative from a set of alternatives considering various existing criteria. Different logical methods are introduced to assist the decision-maker in making the best choice from the alternatives.

One of the most famous methods in this field is Best-Worst method that is first proposed by Rezaei [16]. In this method, we first among all the criteria, denote the best and worst criteria. Then, we compare the other criteria with these two criteria. So that using numbers 1 to 9, we indicate the priority of the best criterion over the other existent criteria (a_{Bj}). Also, the priority of all the criteria over the worst criteria will be indicated (a_{jW}). Finally, the best weights for the criteria will be derived by solving the following mathematical optimization problem (Rezaei [16]).

$$
\begin{aligned}
&\min \max_j \left\{ \left| \frac{w_B}{w_j} - a_{Bj} \right|, \left| \frac{w_j}{w_W} - a_{jW} \right| \right\} \\
&s.t. \quad \textstyle\sum_j w_j = 1 \\
&\qquad\quad w_j \geq 0, \quad \forall j
\end{aligned} \tag{1}
$$

This program can be converted to the following equivalent mathematical one.

$$
\begin{aligned}
&\min \xi \\
&s.t. \ \left| \frac{w_B}{w_j} - a_{Bj} \right| \leq \xi, \quad \forall j \\
&\quad \left| \frac{w_j}{w_W} - a_{jW} \right| \leq \xi, \quad \forall j \\
&\quad \textstyle\sum_j w_j = 1 \\
&\quad w_j \geq 0, \quad \forall j
\end{aligned} \tag{2}
$$

Suppose we have n criteria. By solving model (2) we obtain the optimal weights of the criteria $(w_1^*, w_2^*, \ldots, w_n^*)$. However, the consistency between judgments is an important issue. Our judgments are fully consistent if $a_{Bj} \times a_{jW} = a_{BW}$, for all j. But, in real applications, the judgments are not fully consistent. In this case, Rezaei [16] defined a consistency ratio (CR) to evaluate the consistency of the obtained results. Depending on the amounts of a_{BW}, we have:

$$
Consistency\ Ratio\ (CR) = \frac{\xi^*}{Consistency\ index} \tag{3}
$$

ξ^* is the optimal solution of model (2), and the consistency index is chosen from the following table.

Table 1. Consistency index

a_{BW}	2	3	4	5	6	7	8	9
Consistency index (max ξ)	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

The amounts of the consistency index are the obtained results from solving the following equation for different values of a_{BW} from 2 to 9. (for more details see Rezaei [16]).

$$
\xi^2 - (1 + 2a_{BW})\xi + ({a_{BW}}^2 - a_{BW}) = 0 \tag{4}
$$

The consistency ratio (CR) is between zero and one. Its lower value shows the more consistency of the comparisons and the more trustworthy of the obtained results.

3 Intuitionistic fuzzy Best-Worst method

In real-world applications, we often face non-crisp and uncertain data. Also, there exists a degree of hesitation in the reported data from the decision-maker(s). To model these data, intuitionistic fuzzy numbers (IFNs) can be an appropriate tool.

Here, using intuitionistic fuzzy numbers to report the comparisons of criteria in the Best-Worst method, the model (1) will be converted to the following intuitionistic fuzzy mathematical problem:

$$\begin{aligned} &\min \max_j \left\{ \left| \frac{\widetilde{w}_B}{\widetilde{w}_j} - \tilde{a}_{Bj} \right|, \left| \frac{\widetilde{w}_j}{\widetilde{w}_W} - \tilde{a}_{jW} \right| \right\} \\ &s.t. \quad \textstyle\sum_j \widetilde{w}_j = \tilde{1} \\ &\qquad \widetilde{w}_j \geq \tilde{0}, \quad \forall j \end{aligned} \tag{5}$$

Similarly, model (5) can be converted to the following equivalent mathematical one.

$$\begin{aligned} &\min \xi \\ &s.t. \quad \left| \frac{\widetilde{w}_B}{\widetilde{w}_j} - \tilde{a}_{Bj} \right| \leq \xi, \quad \forall j \\ &\quad \left| \frac{\widetilde{w}_j}{\widetilde{w}_W} - \tilde{a}_{jW} \right| \leq \xi, \quad \forall j \\ &\quad \textstyle\sum_j \widetilde{w}_j = \tilde{1} \\ &\quad \widetilde{w}_j \geq \tilde{0}, \quad \forall j \end{aligned} \tag{6}$$

One of the most famous approaches for solving intuitionistic fuzzy mathematical problems is applying an appropriate transformation index. In this approach, we first assign a crisp number using a meaningful index to each IFNs and then, we solve the converted intuitionistic fuzzy mathematical problem. Recently, Shakouri et al. [25] introduced an appropriate parametric index for IFNs. This index considers the decision maker's idea in the decision process by applying decision-level and hesitation-degree parameters.

In the following, we first briefly introduce this index and then, we apply it to propose a new approach for solving MCDM problems using intuitionistic fuzzy Best-Worst method.

Let $\tilde{I} = (\mu'_{\tilde{I}}(r), v'_{\tilde{I}}(r))$ be an arbitrary IFN (definition 2.9), Shakouri et al. [25] assigned the following index:

$S_{\alpha,k}(\tilde{I}) = \int_{\alpha}^{w} (\underline{P}(r) + \overline{P}(r))dr + k \left[\int_{\alpha}^{1-u} (\underline{Q'}(r) + \overline{Q'}(r))dr - \int_{\alpha}^{w} (\underline{P}(r) + \overline{P}(r))dr \right]$,

$0 \leq \alpha < w, k \in [0,1]$, where $\underline{Q}'(r)$ and $\overline{Q'}(r)$ belong to $1 - v'_{\tilde{A}}(r)$ function.

The parameters α and k will be chosen by the decision-maker. Selecting the parameter α close to one is named a "high-level decision" because just the elements with high membership values in an IFN will be important. "Low-level decision" is made when we set the parameter α close to zero. In this case, the elements of an IFN with low membership values will be considered, too. Also, a pessimistic decision will be made if we set $k = 0$, because the hesitation area of an IFN will not be considered as the membership function in the decision process. Moreover, setting $k = 1$ is named an optimistic decision because all the hesitation areas of an IFN will be considered as the membership function. So, selecting k between zero and one ($0 < k < 1$) reflects a desirable degree between the pessimistic and the optimistic point of view about the hesitation area of an IFN in the decision process.

Let $\tilde{I} = (\mu_I'(r), v_I'(r))$ be a trapezoidal IFN (TraIFN) (figure 3), then we have:

$$S_{\alpha,k}(\tilde{I}) = \int_\alpha^w (\underline{P}(r) + \overline{P}(r))dr + k\left[\int_\alpha^{1-u} (\underline{Q'}(r) + \overline{Q'}(r))dr - \int_\alpha^w (\underline{P}(r) + \overline{P}(r))dr\right]$$
$$= (1-k)\left[(a+d)(w-\alpha) + (b+c-a-d)\left(\frac{w^2-\alpha^2}{2w}\right)\right] + k\left[(e+\mathrm{h})(1-u-\alpha) + (\mathrm{f}+\mathrm{g}-\mathrm{e}-\mathrm{h})\frac{(1-u)^2-\alpha^2}{2(1-u)}\right], \quad 0 \le \alpha < w, k \in [0,1]\,. \tag{7}$$

Shakouri et al. [25] proved that this index has the required properties of a reasonable ranking index proposed by Wang and Kerre [26,27].

Substituting the IFNs (that applied in the Best-Worst method to compare criteria) with their assigned $S_{\alpha,k}$ index, the model (6) will be converted to the following parametric mathematical model which is dependent on the parameters α (decision level) and k (hesitation degree).

$$\begin{aligned} &\min \xi \\ &s.t.\ \left|\frac{w_B}{w_j} - S_{\alpha,k}(\tilde{a}_{Bj})\right| \le \xi, \ \ \forall j \\ &\quad \left|\frac{w_j}{w_W} - S_{\alpha,k}(\tilde{a}_{jW})\right| \le \xi, \ \ \forall j \\ &\quad \textstyle\sum_j w_j = 1 \\ &\quad w_j \ge 0, \ \ \forall j \end{aligned} \tag{8}$$

, where $0 \le \alpha < w^*$ ($w^* = min\left\{w_{\tilde{a}_{Bj}}, w_{\tilde{a}_{jW}}/\forall j\right\}$, and $0 \le k \le 1$.

After determining desirable amounts of the parameters α and k by the decision-maker, the model (8) will convert to a non-linear programming model and be easily solved. In Figure 4, we summarized the steps of the developed Best-Worst method for solving an MCDM problem with intuitionistic fuzzy data.

4 An illustrative example

Here, by an illustrative example, we demonstrate the ability of our proposed method to solve intuitionistic fuzzy MCDM problems using the developed Best-Worst method.

Example. Different factors are important to buy a dream home. Among them, experts participating in our survey chose the following five factors as the more important ones: price (C1), location (C2), quality of building materials and safety (C3), internal space (C4), and access to public transportation (C5). In the Best-Worst method we should first determine the best and the worst factor. Based on the experts' opinion, the quality of building materials and safety (C3) is the best, and the access to public transportation (C5) is the worst factor.

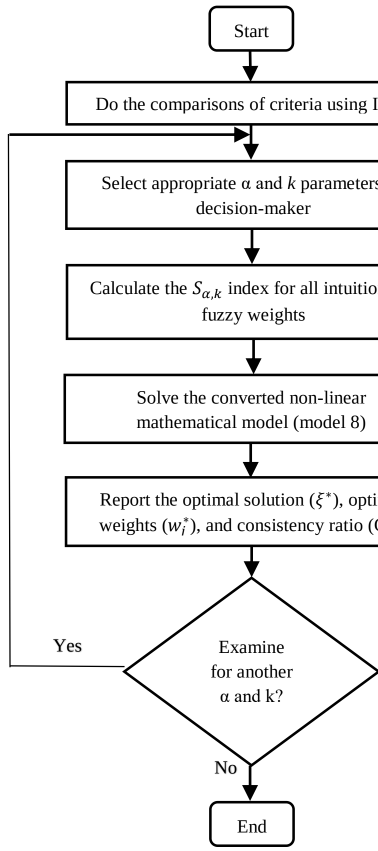

Figure 4. Developed Best-Worst method for solving MCDM problems with intuitionistic fuzzy data.

Then, we should determine the priority of the best criterion over all the other criteria (a_{Bj}), and the priority of all the criteria over the worst criterion (a_{jW}). Table 2 shows the preference evaluations of linguistic descriptions based on Saaty's crisp scale (1/9-9) [20].

Table 2. Evaluation of the linguistic description of the preferences with crisp numbers [20].

Linguistic description of the preferences	Crisp scale
Extremely preferred	9
Very strongly preferred	7
Strongly preferred	5
Moderately preferred	3
Equally preferred	1

Moderately not preferred	1/3
Strongly not preferred	1/5
Very strongly not preferred	1/7
Extremely not preferred	1/9

As mentioned before, IFNs can better model real-world data. Linguistic data are one of the most important and widely used real-world data that are non-crisp and almost always reported with a degree of hesitation. So, to obtain more exact results, we allow the experts to report the comparison results by IFNs ($\tilde{\frac{1}{9}}$-$\tilde{9}$). Tables 3 and 4 show the final comparison results based on the consensus of the experts.

Table 3. Priority of the best criterion (C3) versus all criteria (a_{Bj}).

The best criterion v. all criteria	Quality of building materials and safety (C3)
Price (C1)	$\tilde{6} = \langle(5,6,6,8;1),(4,6,6,8;0)\rangle$
Location (C2)	$\tilde{3} = \langle(2,3,3,4;1),(2,3,3,5;0)\rangle$
Internal space (C4)	$\tilde{4} = \langle(3,4,4,5;1),(3,4,4,7;0)\rangle$
Access to public transportation (C5)	$\tilde{8} = \langle(5,7,9,10;1),(4,6.5,9.5,11;0)\rangle$

Table 4. Priority of all criteria versus the worst criterion (C5) (a_{jW}).

All criteria v. the worst criterion	Access to public transportation (C5)
Price (C1)	$\tilde{2} = \langle(1,2,2,4;1),(1,2,2,4;0)\rangle$
Location (C2)	$\tilde{5} = \langle(3.5,5,5,6;1),(3,5,5,7;0)\rangle$
Quality of building materials and safety (C3)	$\tilde{8} = \langle(5,7,9,10;1),(4,6.5,9.5,11;0)\rangle$
Internal space (C4)	$\tilde{4} = \langle(3,3.5,4.5,5;1),(2.5,3.5,5,5.5;0)\rangle$

After calculating the $S_{\alpha,k}$ index for the intuitionistic fuzzy numbers of table 3, we have:

$S_{\alpha,k}(\tilde{6}) = (1-k)\left[13(1-\alpha)-\left(\frac{1-\alpha^2}{2}\right)\right]+k[12(1-\alpha)]$,

$S_{\alpha,k}(\tilde{3}) = (1-k)[6(1-\alpha)]+k\left[7(1-\alpha)-\left(\frac{1-\alpha^2}{2}\right)\right]$,

$S_{\alpha,k}(\tilde{4}) = (1-k)[8(1-\alpha)]+k\left[10(1-\alpha)-2\left(\frac{1-\alpha^2}{2}\right)\right]$,

$S_{\alpha,k}(\tilde{8}) = (1-k)\left[15(1-\alpha)+\left(\frac{1-\alpha^2}{2}\right)\right]+k\left[15(1-\alpha)+\left(\frac{1-\alpha^2}{2}\right)\right]$, where $0 \le \alpha < 1, k \in [0,1]$.

Table 5. $S_{\alpha,k}$ index of the data in table 3 for different amounts of α and k.

α	k	$S_{\alpha,k}(\tilde{6})$	$S_{\alpha,k}(\tilde{3})$	$S_{\alpha,k}(\tilde{4})$	$S_{\alpha,k}(\tilde{8})$
0.1	0.1	11.16	5.44	7.28	13.99
0.1	0.5	11	5.60	7.60	13.99
o.1	0.9	10.84	5.76	7.93	13.99
0.5	0.1	6.11	3.01	4.02	7.87
0.5	0.5	6.06	3.06	4.12	7.87
0.5	0.9	6.01	3.11	4.22	7.87
0.7	0.1	3.64	1.80	2.41	4.75

0.7	0.5	3.62	1.82	2.44	4.75
0.7	0.9	3.60	1.84	2.48	4.75

Also, for the intuitionistic fuzzy numbers of table 4, we have:

$S_{\alpha,k}(\tilde{2}) = (1-k)\left[5(1-\alpha) - \left(\frac{1-\alpha^2}{2}\right)\right] + k\left[5(1-\alpha) - \left(\frac{1-\alpha^2}{2}\right)\right]$,

$S_{\alpha,k}(\tilde{5}) = (1-k)\left[9.5(1-\alpha) + 0.5\left(\frac{1-\alpha^2}{2}\right)\right] + k[10(1-\alpha)]$,

$S_{\alpha,k}(\tilde{8}) = (1-k)\left[15(1-\alpha) + \left(\frac{1-\alpha^2}{2}\right)\right] + k\left[15(1-\alpha) + \left(\frac{1-\alpha^2}{2}\right)\right]$,

$S_{\alpha,k}(\tilde{4}) = (1-k)[8(1-\alpha)] + k\left[8(1-\alpha) + 0.5\left(\frac{1-\alpha^2}{2}\right)\right]$, where $0 \le \alpha < 1, k \in [0,1]$.

Table 6. $S_{\alpha,k}$ index of the data in table 4 for different amounts of α and k.

α	k	$S_{\alpha,k}(\tilde{2})$	$S_{\alpha,k}(\tilde{5})$	$S_{\alpha,k}(\tilde{8})$	$S_{\alpha,k}(\tilde{4})$
0.1	0.1	4	8.82	13.99	7.22
0.1	0.5	4	8.90	13.99	7.32
o.1	0.9	4	8.98	13.99	7.42
0.5	0.1	2.12	4.94	7.87	4.02
0.5	0.5	2.12	4.97	7.87	4.09
0.5	0.9	2.12	4.99	7.87	4.17
0.7	0.1	1.245	2.98	4.755	2.41
0.7	0.5	1.245	2.99	4.755	2.46
0.7	0.9	1.245	3	4.755	2.51

Tables 5 and 6 show the results of calculating the $S_{\alpha,k}$ index for the data in tables 3 and 4 in some selected amounts of the decision level (α) and the hesitation degree (k).

Now, by the data from tables 5 and 6, we solve the converted non-linear mathematical model (model 8) for different amounts of the parameters α (decision level) and k (hesitation degree) with GAMS software. The results are in table 7. As mentioned before, the appropriate selection of the parameters α and k depends on the decision maker's opinion. From Table 7, we can see that different α and k lead to different optimal weights for the criteria.

Table 7. Optimal weights and consistency ratio.

(α,k)	w_1^*	w_2^*	w_3^*	w_4^*	w_5^*	CR
(0.1,0.1)	0.068	0.227	0.543	0.130	0.032	0.157
(0.1,0.5)	0.071	0.229	0.543	0.126	0.031	0.167
(0.1,0.9)	0.074	0.231	0.543	0.122	0.031	0.176
(0.5,0.1)	0.089	0.212	0.483	0.162	0.054	0.084
(0.5,0.5)	0.069	0.214	0.497	0.165	0.055	0.089
(0.5,0.9)	0.095	0.237	0.464	0.153	0.051	0.096
(0.7,0.1)	0.105	0.228	0.403	0.182	0.082	0.022
(0.7,0.5)	0.105	0.226	0.405	0.182	0.081	0.026

(0.7,0.9)	0.106	0.223	0.407	0.183	0.081	0.031

It is worthwhile to mention that although we have an unchanged ranking result for different amounts of the parameters ($w_3^* > w_2^* > w_4^* > w_1^* > w_5^*$), different obtained values for these weights can be lead to various optimal decisions in the MCDM problems under intuitionistic environments.

5 Conclusion

The intuitionistic fuzzy Multi-criteria decision-making models are an efficient idea to model real-world decision-making problems with multiple criteria. In these models, after assessing the real-world data (that are often non-crisp and reported with a degree of hesitation) with the intuitionistic fuzzy numbers, we should apply an appropriate method to solve them.

In this paper, we propose a new yet effective method for solving intuitionistic fuzzy multi-criteria decision-making problems. In our proposed approach, two significant factors in intuitionistic fuzzy sets, i.e. decision-level (α) and hesitation degree (k) parameters, are controllable in the decision process by the decision-maker. In this way, the calculated final solution is not unique and is dependent on the selected parameters α and k. This is a valuable feature because we expect that the obtained result from solving an intuitionistic fuzzy model be in intuitionistic or at least parametric form. Also, by an illustrative example, we demonstrate how the method works.

Here, we developed our approach for the intuitionistic fuzzy Best-Worst method. But, it can be easily generalized to the other intuitionistic MCDM methods, too.

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